The 7th Bremen Summer School and Symposium

Dynamical Systems - pure and applied

August 5-9, 2019
Faculty of Mathematics
University of Bremen

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Dynamical cross-diffusion systems: modeling, analysis, numerics

Ansgar Jüngel[*,1]

Abstract.

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Course: Mon, 10:10-11:00; Tue, 10:10-11:00; Wed, 9:00-9:50

Ergodic Theorems

Tanja Eisner[*,1]

Abstract.

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Talk: Mon, 14:00-14:30

Mixing for Z Extensions of Gibbs Markov semiflows

Dalia Terhesiu*,[1]

Abstract.

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Talk: Mon, 14:40-15:30

Game theory for microbe population dynamics

Julie Rowlett[*,1]

Abstract.

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Talk: Mon, 16:00-16:30

Strong laws under trimming - a comparison between iid random variables and ergodic transformations

Tanja Schindler*,[1]

Trimming, i.e. removing the largest entries of a sum of iid random variables, has a long tradition to prove limit theorems which are not valid if one considers the untrimmed sum - for example a strong law of large numbers for random variables with an infinite mean.

For certain ergodic transformations, for example piecewise expanding interval maps, and certain observables over those transformations the results are essentially the same as in the iid case. However, considering the same ergodic transformation and an observable with a different distribution function, the system can behave completely different to its iid counterpart. I will give an overview of some of the (sometimes surprising) trimming results. This is partly joint work with Marc Kesseböhmer.

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Talk: Mon, 16:40-16:55

On the possibility of a growth gap for counting periodic orbits in non-compact covers

Rhiannon Dougall*,[1]

Let M be a compact negatively curved manifold. The "number of closed geodesics of length less than T" grows exponentially in T, and the exponential growth rate $h(M)$ is "the entropy of the system". Suppose that M' is a non-compact regular cover of M. We count closed geodesics that intersect some compact set, and get an associated growth rate $h(M')$. We ask, is it possible that there is some δ (uniform over all regular covers M') so that either $h(M') = h(M)$ or $h(M') < h(M) - \delta$? We are able to characterise the possibility of such a gap. For instance, if the fundamental group of M has Property (T) then there is a gap in the growth rates. There are also results for countable group extensions of subshifts of finite type.

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Talk: Mon, 17:00-17:15

On Non-Linear Pseudo Anosov map and parabolic flows

Paolo Giulietti*,[1]

I will present a work-in-progress which focuses on ergodic averages and cohomological equations related to parabolic flows. In a nutshell, I will show a strategy to renormalize such flows on surfaces with singularities by nonlinear pseudo Anosov transformations. This approach allows us to exploit transfer operator techniques on anisotropic Banach spaces to control averages. Joint work with M. Artigiani.

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Talk: Tue, 14:00-14:30

Connecting dynamical systems and inverse spectral theory

Martina Chirilus-Bruckner[*,1]

The construction of breather solutions for a nonlinear wave equation with periodic coefficients can be reformulated via dynamical systems techniques (such as center manifold reduction) as an inverse spectral problem. After briefly sketching such a reformulation we explain preliminary results, as well as new possibilities and challenges of this endeavor.

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Talk: Tue, 14:40-15:30

Strichartz estimates and attractors for dispersive-dissipative PDEs

Sergey Zelik[*,1]

Abstract.

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Talk: Wed, 14:00-14:30

The dynamics of flipped alpha-continued fractions

Charlene Kalle[*,1]

Abstract.

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Talk: Wed, 14:40-15:30

Computing Ruelle resonances of chaotic dynamical systems

Oscar Bandtlow *,[1]

Abstract.

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Public Lecture: Wed, 18:00-19:00

Paradoxical decompositions, groups and growth rates

Richard Sharp[*,1]

The Banach-Tarski paradox says that it is possible to decompose a 3-dimensional ball into a finite number of pieces and rearrange them to form two copies of the original ball. This is perhaps the most striking example of the type of paradoxical decompositions that were discovered in the early 20th century. It was already realised in the 1920s that these decompositions are intimately related to the structure of the underlying symmetry groups. Perhaps even more surprising, the last 60 years have seen that this theory is related to an apparently very different set of problems: understanding some of the growth (or decay) rates that occur in probability, geometry and chaotic dynamics. I will discuss some of these topics and the connections between them.

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Course: Thu, 9:00-9:50, 10:10-11:00; Fri, 10:10-11:00

Topics in Thin Groups

Alex Kontorovich[*,1]

Abstract.

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Talk: Thu, 14:00-14:30

Spectral gap for coupling of random walks on compact groups

Keivan Mallahi-Karai[*,1], Amir Mohammad [2], Alireza Salehi Golsfeld [2]

Let G_1 and G_2 be either compact simple groups, and let μ_1 and μ_2 be symmetric probability measures on G_1 and G_2, respectively. Under some mild conditions on μ_1, μ_2, one knows that the distribution of the random walk on G_i driven by μ_i converges to the uniform distribution, and the speed of convergence is governed by the spectral gap of μ_i.

A coupling of μ_1 and μ_2 is a probability measure μ on $G_1 \times G_2$ with marginal distributions μ_1 and μ_2, respectively. Under what conditions does μ have a spectral gap depending on the gaps of μ_1 and μ_2?

In this talk I will first review some of the old and new methods for establishing spectral gaps, mainly based on pioneering work of Bourgain-Gamburd and then discuss the question stated in the previous paragraph.

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Talk: Thu, 14:40-15:30

Dynamical renewal functions

Sabrina Kombrink[*,1]

Abstract.

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Talk: Thu, 16:00-16:30

Large fronts in nonlocally coupled systems using Conley-Floer homology

Bente Bakker *,[1]

This talk is on travelling front solutions for equations of the type

$$\partial_t u = N * S(u) + \nabla F(u), \quad u(t,x) \in \mathbf{R}^d.$$

Here $N*$ denotes a convolution-type operator in the spatial variable $x \in \mathbf{R}$, either continuous or discrete. We develop a novel Morse-type theory, the Conley-Floer homology, which captures travelling front solutions in a topologically robust manner, by encoding fronts in the boundary operator of a chain complex. In various cases the resulting Conley-Floer homology can be interpreted as a homological Conley index for multivalued vector fields. Using the Conley-Floer homology existence and multiplicity results on travelling front solutions are derived.

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Talk: Thu, 16:40-16:55

Resonances for Schottky surfaces

Alexander Weiße [*,1]

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[1]Max Planck Institute for Mathematics Bonn, Germany

Talk: Fri, 11:10-12:00

Periodic orbit growth on covers of Anosov flows

Richard Sharp[*,1], Rhiannon Dougall [1]

It is well-known that the topological entropy of an Anosov flow on a compact manifold describes the exponential growth rate of its periodic orbits. If we pass to a regular cover of the manifold then we can consider a corresponding growth rate for periodic orbits of the lifted flow. This growth rate is bounded above by the original topological entropy but if the cover is infinite then the growth rate may be strictly smaller. In the important special case of a geodesic flow over a compact manifold with negative sectional curvatures, we have equality if and only if the cover is amenable (Dougall & Sharp, Math. Annalen, 2016). This result fails for general Anosov flows but we will discuss a recent result that gives a natural generalisation.

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4 Contributed Talk Abstracts

Synchronization in the networks of the second-order Kuramoto oscillators with time-varying natural frequencies

Nikita Barabash*,[1], Vladimir N. Belykh [1]

In the theory of evolving dynamical networks, the Kuramoto model in its non-stationary variations (with plasticity of connections [**1**, **2**], with switchable topology [**3**], with time-varying natural frequencies [**4**]) remains a popular example. However, despite the large number of papers devoted to this model the theory lacks of rigorous analytical results. In this talk, we present a rigorous proof of the global synchronization existence in the network of N coupled second-order Kuramoto oscillators

$$\beta\ddot{\varphi}_i + \dot{\varphi}_i = \omega_i(t) + \frac{K}{N}\sum_{j=1}^{N}\sin(\varphi_j - \varphi_i), \quad i = 1, 2, \ldots, N,$$

where a positive parameter β represents an inertia of i-th oscillator, $\omega_i(t)$ is a time-varying natural frequency of i-th oscillator, and a parameter K is a coupling strength. We state our main result as follows.

Theorem 1 *Let for* $K = 1$, $|\omega_i(t) - \omega_N(t)| < 2\cos\frac{3\alpha}{2}\sin\frac{\alpha}{2}$, $i = 1, 2, \ldots, N$, $\beta < (4\cos\frac{3\alpha}{2})^{-1}$, *where* $\alpha < \pi/3$. *Then the global synchronization with the accuracy* $|\varphi_i - \varphi_N| < \alpha$ *exists in the system.*

In our talk, we provide the proof of this statement as well as numerical validation.

This work was supported by the Russian Foundation for Basic Research (the project No. 18-01-00556, analytical results) and the

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Russian Science Foundation (the project No. 19-12-00367, numerical results).

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Homoclinic orbits at bifurcations in the Circular Restricted Four Body Problem

Wouter Hetebrij[*,1], Jason Mireles James [2]

In the Circular Restricted Four Body Problem, we have 3 bodies in an equilateral configuration and are interested in the movement of a fourth massless body. Depending on the masses of the first three objects, there are 8, 9 or 10 Lagrangian points for the fourth body. The bifurcation diagram exists of a one-dimensional curve of Hamiltonian saddle-node bifurcations and single point where there is a subcritical Hamiltonian pitchfork bifurcation.

If there are 10 Lagrangian points, there exist homoclinic orbits to some of the stationary points of the fourth body. In this talk, we will prove that the homoclinic orbit persists at the Hamiltonian saddle-node bifurcations. We compute the homoclinic orbit in two steps.

The first step is finding a local parameterization of the two-dimensional center manifold at the bifurcation point together with its dynamics. Using the conjugate dynamics on the center manifold, we show the existence of one-dimensional stable and unstable submanifolds inside this center manifold. The second step is computing the global stable and unstable manifolds and finding an intersection of the two global manifolds.

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Optimal balance for geophysical flows and spontaneous wave emission

Gökce Tuba Masur[*,1], Marcel Oliver [1]

To provide full understanding of a dissipation route of energy in the ocean, it is necessary to decompose geophysical flows into balanced and imbalanced components due to nonlinear coupling between the components. We know the method of optimal balance used for flow decomposition, which is introduced by Viúdez and Dritschel (2004) in the context of rapidly rotating fluid flow with Langrangian view to provide balanced initializations for geophysical flows. We, however, still do not know about quantification of imbalanced flows starting with a balanced initialization and the importance of these flows in the route to dissipation of energy. To analyse them, it is necessary to perform diagnostic derivation of balanced flows from geophysical ocean models in primitive variables. We, therefore, aim to investigate optimal balance in terms of primitive variables for the two-dimensional rotating shallow water model on f-plane.

In our method, the decomposition of balanced-imbalanced flows is carried out through adiabatically deforming the nonlinear rotating shallow water model into a linear one for which mode-splitting is exact, where this procedure is treated as a boundary value problem in time to be solved iteratively until converging a balanced flow. In the model implementation, all dynamics part are formulated in primitive variables while appearance of a kinematic potential vorticity inversion is inevitable owing to having robust demonstration of the method only if potential vorticity is used as a "base-point field". After the optimal balance algorithm set-up, our search is currently moving to apply the algorithm to different scenarios and observe reasonable convergence to a balanced flow where small excitation of imbalances follows.

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On Hausdorff dimension of thin nonlinear solenoids

Reza Mohammadpour [*,1], Feliks Przytycki [1], Michał Rams [1]

Let $M = S^1 \times \mathbb{D}$ be the solid tours, where $S^1 = \frac{\mathbb{R}}{2\pi\mathbb{Z}}, \mathbb{D} = \{v \in \mathbb{R}^2 | |v| < 1\}$ carries the product distance $d = d_1 \times d_2$ and suppose $S : M \to M$ such that

$$(x, y, z) \mapsto (\eta(x, y, z) \mod 2\pi, \lambda(x, y, z), \mu(x, y, z))$$

is a smooth embedding map where η, λ and μ are close to constant. Bothe [**1**] was the first who obtained results on the dimension of the attractor of a thin linear solenoid where contraction rates are strong enough. Barriera, Pesin and Schemeling [**2**] established a dimension product structure of invariant measures in the course of proving the Eckmann Ruelle conjecture.

Conjecture : The fractal dimension of a hyperbolic set is (at least generically or under mild hypotheses) the sum of those of its stable and unstable slices, where fractal can mean either Hausdorff or upper box dimension.

In spite of the difficulties due to possible low regularity of the holonomies, indeed, Hasselblatt and Wilkinson [**4**] found open sets of symplectic Anosov maps with the property that on a residual set of full measure (with respect to any invariant measure) the subbundles are not Lipschitz, and the holonomies are non-Lipschitz a.e. with respect to Lebesgue measure. Hasselblatt and Schmeling [**3**] have proved the conjecture for a class of thin linear solenoids. We prove the conjecture for a class of thin nonlinear solenoids.

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[3] B. Hasselblatt and J. Schmeling, Dimension product structure of hyperbolic sets, Modern Dynamical Systems and Applications, Cambridge University Press, New York, 331-345, 2004.

[4] B. Hasselblatt and A. Wilkinson. Prevalence of non-Lipschitz Anosov foliations, Ergodic Theory and Dynamical Systems, 19: 643-656, 1999.

Diffusion on dynamical interbank loans networks

Nikolaos Poulios*,[1]

This subject of leverage in financial networks is among the basics so as to calculate the robustness of the network. Here, in our study we use differential equations. In our research we are studying the dynamics of interbank financial networks, through the flow of capitals, leverage balancing and the stability of the network.

In the endeavor of this whole study we focus on connected and directed graphs. These graphs have some features, such as the nodes, which in our study are the banks and which therefore will be so many, as the number of total banks in any case. Another important feature is the edges, which gives the information concerning which bank provides another one with a loan. We get this information if we draw an arrow at the end of every edge and by doing this we get the direction of the way loans move from one bank to the other. In those graphs there are some fundamental tools which we are using throughout our research and these are the Adjacent, Incident and Laplacian matrices, respectively. Having all those tools we build a system of differential equations. The solution of differential equations is of the form $\varphi(t) = e^{\mathbf{A}t}\varphi(0)$ and a very important role has the spectral analysis of operator $\mathbf{A}$. Moreover, we study the differential equations and their operators in order for them to be applied in understanding the financial network stability.

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Transitions in Dynamical Systems with Bounded Uncertainty

Kalle Timperi*,[1]

As an alternative to stochastic differential equations, the assumption of bounded noise can offer a flexible and transparent paradigm for modelling systems with uncertainty. In particular, it offers an avenue for carrying out stability and bifurcation analysis in random dynamical systems, using a set-valued dynamics approach. The system under study is assumed to be a discrete time dynamical system in a low-dimensional Euclidian space, with bounded random kicks at each time step. The collective behaviour of all future trajectories is then represented by a set-valued map, whose minimal invariant sets represent the stable state-space regions of the system, including the effect of the noise/randomness.

We describe in the 2-dimensional case the geometric properties of minimal invariant sets, and provide a classification result for the singularity points on their boundaries. The geometry is quite well understood in this case, but becomes increasingly more complicated in higher dimensions. The geometric picture obtained serves as a stepping stone for further dynamical analysis of the systems.

In order to illuminate the loss of stability near a set-valued bifurcation, we will discuss the boundary dynamics of the minimal invariant sets, and present a numerical scheme for tracking the boundaries for changing parameter values and noise amplitudes.

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5 Poster Abstracts

Fixed point theorem for delay dynamic equations on time scalese

Kamel Ali Khelil*,[1], Abdelouaheb Ardjouni [2]

In this work, based on the theory of calculus on time scales, we use the Krasnoselskii-Burton's fixed point theorem to obtain asymptotic stability and stability results about the zero solution for the nonlinear delay dynamic equations on time scales. These results have important leading significance in various areas of science and engineering.

Keywords: Fixed point theorem, dynamic equations, time scales, stability.
2010 Mathematics Subject Classification: 34K20, 34N05, 45J05.

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Applying of 1D mappings to study Shilnikov spiral attracors in strongly dissipative three-dimensional flows

Yuliya Bakhanova*,[1]

In this work, we present some new results of the study of bifurcation set near the Shilnikov homoclinic loop of a saddle-focus equilibrium in the Rossler [1] and Rosenzweig-MacArtur systems [2]. In the parameter plane of these systems, the so-called U-shaped bifurcation curve corresponds to a homoclinic loop of a saddle-focus with one-dimensional stable and two-dimensional unstable invariant manifolds. Such a bifurcation curve consists of two branches the distance between which is very small (10^{-10}) [2,3]. These two branches are converged in the specific 2-codimensional point, which is called as the periodicity hub [3].

In [3] it is shown that along the homoclinic bifurcation curve there is an infinite number of additional periodicity hubs. Using one-dimensional maps we explain global organizing of periodicity hubs emerging along the homoclinic bifurcation curve for strongly dissipative systems possessing the Shilnikov spiral attractor. We explain the nature of secondary homoclinic bifurcation curves emerging near each such hubs. Using one-dimensional maps we also show how to predict the form of secondary homoclinic orbits. Finally, by explicit constructing homoclinic orbits, we confirm our prediction.

This work was supported by the RFBR grant No. 18-31-00431.

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Physical Review E, 2011, Vol. 84, No. 3, p. 035201.

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Physical Review E, 2011, v.84(1), p. 016216.

On the boundary between Lorenz attractor and quaisattractor in Shimizu-Morioka system

Andrey Bobrovskiy*,[1], Alexey Kazakov [1], Ivan Korenkov [1], Klim Safonov [1]

We study the boundary between Lorenz attractor and qusiattractors in the Shimizu-Morioka system [1]

$$\begin{cases} \dot{x} = y, \\ \dot{y} = x - \lambda y - xz, \\ \dot{z} = -\alpha z + x^2. \end{cases}$$

Here x, y, and z are phase variables, $\alpha > 0$ and $b > 0$ – parameters of the system.

Under Lorenz attractor, we mean the stable closed invariant set which satisfies the condition of the geometrical model of Afraimovich, Bykov, Shilnikov [2]. It means that it has a pseudohyperbolic structure and, thus, it is robust with respect to small changing in parameters. The class of quasiattractors was introduced by Afraimovich and Shilnikov and contains non-robust strange attractors which either possess stable periodic orbits with large periods and narrow absorbing domains or such orbits appear with arbitrarily close perturbation [3, 4].

In the classical Lorenz system the boundary between Lorenz attractor and quasiattractors is formed by the curve $l_{A=0}$ where the separatrix value A of the corresponding Poincaré maps vanishes [5]. On the one side from the curve $l_{A=0}$, when $A > 0$, the attractor is pseudohyperbolic, and it becomes a quaisattractor on the other side, when $A < 0$. The violating of pseudohyperbolicity on the curve $l_{A=0}$ is associated with the destruction of the stable foliations in the corresponding Poincare map [2]. It is imortant to note, that in the Lorenz system the saddle index ν of the saddle equilibrium $O(0, 0, 0)$ is less than $1/2$ along the part of curve $l_{A=0}$ bounded the region of existence of the Lorenz attractor.

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In the Shimizu-Morioka system, the saddle index ν along the curve $l_{A=0}$ belongs to the interval $\nu \in [\nu_1, \nu_2]$, where $\nu_1 \approx 0.31$ and $\nu_2 \approx 0.82$. Thus, the boundary between Lorenz attractor and quasi-attractor is much more complicated. For $\nu \leq 1/2$ it coincide with the curve $l_{A=0}$ as in the classical Lorenz model. However, in the case $\nu > 1/2$, stable periodic orbits appear in the system even for positive values of the separatrix value near the curve $l_{A=0}$. Hence the region of existence of the Lorenz attractors in this case is formed by the upper boundary of the corresponding stability windows.

For the detailed analysis of bifurcations in the neighborhood of the curve $l_{A=0}$ we study a one-dimensional factor-map of the corresponding Poincare map

$$\bar{x} = (-1 + A|x|^\nu + B|x|^{2\nu}) \cdot sign(x).$$

We establish that our theoretical investigations are in the full agreement with the numerical study of the boundary of the existence of Lorenz attractor in the Shimizu-Morioka model.

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Random Multifractals and Fractals generated by Ergodic Processes

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By means of the large deviation principle we study the multifractal analysis in a random environment.That is to say, we study the Hausdorff dimension of level sets of certain random functions with respect to the random metric in the context of random homogeneous fractal.

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Bifurcation analysis of a neuralnetwork in the olfactory bulb using equation-free methods

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The olfactory bulb's (OB) neural network with individually firing neurons in the olfactory bulb, which is responsible for odour recognition, is a good model system to study the brain's performance because of its well-defined input and output.

For the direct interaction among cells in high-dimensional biological neural networks exist detailed models. We use a Spike and Response Model (SRM [1,2]) to simulate numerically the OB. If one considers the distinction of two different odours, both, biological measurements [3] in mammals and direct simulation, show the same macroscopic behaviour. If one odour is dominant and the concentration ration for the two odours is slightly changed, the first one's respective area is no longer dominantly active as if the other odour was first dominant. Therefore, we consider the difference of the fire rates of their different appendant areas as the macroscopic variable. Hence, the detection of an odour depends on the fire rates in different areas of the neural network. By direct simulation, one can already get the stable branches of the fire rates. In an odour concentration/fire rate difference-diagram, hysteresis behaviour can be observed.

In order to track the unstable branches by implicit equation-free methods [4, 5], we use an altered Newton method, which deals with noisy derivative information due to the quasi-chaotic fire rates close to equal concentration.

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On the region of existence of discrete Lorenz attractor in a nonholonomic Celtic stone model

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We consider the problem on the existence of discrete Lorenz attractors in a nonholonomic Celtic stone model. To this end, in two-parameter families of such models of certain types, the main local and global bifurcations leading to both the appearance and destruction of the attractors are studied. In the plane of governing parameters (one of them is the angle of dynamical asymmetry of the stone, and the other is the total energy), we construct the corresponding bifurcation diagram, where the region of existence of the discrete Lorenz attractor is shown and its boundaries are explained. We point out the similarities and differences in the scenarios of the emergence of the discrete Lorenz attractor in the nonholonomic model of Celtic stone and the attractor from the classical Lorenz model.

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About two-dimensional diffeomorphisms with a quadratic homoclinic tangency to a nonhyperbolic saddle

Olga Gordeeva*,[1]

We consider a C^r-diffeomorphism $f_0 (r \geq 4)$, satisfying the following conditions:

(A) f_0 has a fixed point O that is a nonhyperbolic saddle with multipliers $\lambda_1 = \lambda$, where $o < |\lambda| < 1, \lambda_2 = 1$, and with the second Lyapunov value $l_3 > 0$.

(B) Invariant manifolds $W^u(O)$ and $W^{ss}(O)$ of O have a (single-round) quadratic tangency at the points of a homoclinic orbit Γ_0.

Conditions A) - B) define a codimension 3 locally connected bifurcation surface in the space of two-dimensional diffeomorphisms, and, hence, for studying bifurcations of f_0 we must consider 3-parameter families. Let f_μ, where $\mu = (\mu_1, \mu_2, \mu_3)$, be such a family which unfolds generally degenerations given by conditions A) - B).

Theorem 1 *Let U be a sufficiently small neighborhood of the origin in the $\mu = (\mu_1, \mu_2, \mu_3)$-parameter space. Then in U there is a two-dimensional discontinuous surface $S(\mu)$ (with the edge on the curve $\Gamma_1^* : \mu_3 = \sqrt[3]{\frac{\mu_1}{2}}, \mu_2 = -3\sqrt[3]{\left(\frac{\mu_1}{2}\right)^2}$) such that f_u has a (single-round) quadratic homoclinic tangency either to a hyperbolic saddle fixed point or to a saddle-node fixed point when $\mu \in \Gamma_1^*$.*

Theorem 2 *In the (μ_1, μ_2, μ_3) parameter space, in any sufficiently small neighborhood $U(\mu = 0)$ there are infinitely many nonintersecting domains Δ_k such that at $\mu \in \Delta_k$ the diffeomorphism f_u has asymptotically stable single-round periodic orbit.*

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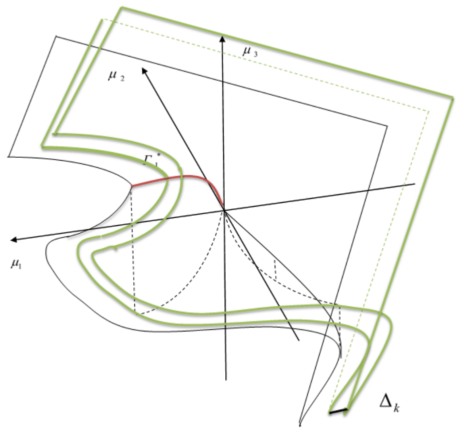
1. Boundaries of Δ_k correspond to codimension 1 bifurcations for single-round periodic orbits - saddle-node and period doubling ones.

2. Domains Δ_k accumulate to the surface $S(\mu)$ as $k \to +\infty$



On pseudohyperbolic attractors, quasiattractors and their examples

Alexey Kazakov*,[1]

In accordance with "P or Q" hypothesis proposed in [1] all chaotic attractors can be divided into two classes: pseudo hyperbolic attractors and quasi attractors.

Every orbit in the pseudohyperbolic attractor is unstable (i.e. it has a positive maximal Lyapunov exponent). Moreover, this instability property persists for all small perturbations of the system. The detailed definition of pseudohyperbolic attractors can be found in [1]. The class of such attractors contains uniformly hyperbolic attractors (e.g. solenoid of Smale-Williams, Plykin's attractor); singular-hyperbolic (Lorenz-like) attractor; and wild spiral attractors of Turaev-Shilnikov type [2]. The first example of such attractors was found quite recently in the four-dimensional Lorenz model [1].

Concerning quasiattractors [3, 4], they either contain stable periodic orbits (with very narrow absorbing domains) or such orbits appear for arbitrarily small perturbation. From the physical point of view, such attractors in many cases do not differ from the pseudohyperbolic ones. However, in the case of quasiattractors, one can observe a chaotic behavior (with positive Lyapunov exponent) but one can never be sure that increasing the accuracy of the computation time would not make the maximal Lyapunov exponent vanish. Examples of the Afraimovich-Shilnikov quasiattractors are numerous. They include "torus-chaos" attractors arising after the breakdown of two-dimensional tori and after a period-doubling cascade, the Henon-like attractors, attractors in periodically perturbed two-dimensional systems, attractors in the Lorenz model beyond the boundary of the region of Lorenz attractor existence, spiral attractors in three-dimensional systems with a Shilnikov loop etc.

In this work, we show examples of pseudohyperbolic attractors and quasiattractors of different types and also present the methods of

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verification of the pseudohyperbolicity of attractors [1, 5].

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Gromov Hyperbolic Graphs for Infinite Iterated Function Systems

Christina Moor*,[1]

We construct a non-locally finite rooted graph for an infinite iterated function system of similarity mappings on $\mathbb{R}^d$ following the ideas from Lau–Wang (2017). In particular, this graph is hyperbolic in the sense of Gromov. The visual boundary, that is the space of the equivalence classes of infinite geodesics starting at the root, is a subset of the Gromov boundary. A coding map establishes a Hölder equivalence of the visual boundary and the limit set of the infinite iterated function system.

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Birth of discrete Lorenz attractors in global bifurcations

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Discrete Lorenz attractors are chaotic attractors, which are the discrete-time analogues of the well-known continuous-time Lorenz attractors. They are genuine strange attractors, i.e. they do not contain simpler regular attractors such as stable equilibria, periodic orbits etc. In addition, this property is preserved under small perturbations. Thus, Lorenz attractors, discrete and continuous, represent the so-called robust chaos.

I present a list of global (homoclinic and heteroclinic) bifurcations [**1**, **2**, **3**, **4**], in which it was possible to prove the appearance of discrete Lorenz attractors. The proof is based on the study of first return (Poincaré) maps, which are defined in a small neighbourhood of the homoclinic or heteroclinic cycle. The first return map can be transformed to the form asymptotically close to the three-dimensional Hénon map via smooth transformations of coordinates and parameters. According to [**1**, **5**, **6**, **7**], Hénon-like maps possess the discrete Lorenz attractor in an open subset of the parameter space.

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Bifurcation analysis of two-dimensional dissipative cubic Henon maps

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In this work, we present some results of bifurcation analysis of one- and two-dimensional cubic Henon map in dissipative case. Also, we pay special attention to the study of the transition from regular dynamics (asymptotically stable periodic orbits) to strange attractors in these systems.

Strange attractors in these maps appear due to a cascade of period-doubling bifurcation with the followed by a cascade of heteroclinic bifurcation leading to bands merging. Also, we note that all strange attractors that we observe in one- and two-dimensional cubic Henon map belong to a class of quasiattractors introduced by Afraimovich and Shilnikov in [1]. Such attractors either contain stable periodic orbits (with very narrow absorbing domains) or such orbits appear for arbitrarily small perturbation. On the bifurcation diagrams for systems with quasiattractors regions with chaotic dynamics alternate with the so-called stability windows inside which asymptotically stable periodic orbits exist.

We pay attention that the stability windows inside regions with chaotic behavior for the one- and two-dimensional cubic maps look like stability windows which appear for strange attractors observed in more complex models, e.g. in Rossler and Rosenzweig-Macartur systems demonstrating Shilnikov spiral attractors. In future work, we plan to study the structure and bifurcations inside stability windows in one- and two-dimensional cubic maps in details. This study will help to understand the nature of strange quasiattractors attractors in more complex systems.

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